

# Overview of the theory of $W/Z$ + jets and heavy flavor

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I review the status of theoretical predictions for events containing a  $W$  or  $Z$  boson and jets, one or more of which may include heavy quarks. Special attention is paid to comparisons between different theoretical approaches and with the latest experimental data.

## 1. INTRODUCTION

$W$  and  $Z$  bosons are currently observed at a very high rate at the Tevatron, with cross sections of the order of a nanobarn, and will be even more copiously produced at the LHC. In such hadronic environments, the vector bosons are observed in association with strongly-interacting particles, which are understood in terms of the radiation of additional gluons and quarks in Quantum Chromodynamics (QCD). Much of this radiation is *soft*, corresponding to transverse energy deposits of a few GeV or less. However, producing a significant deposit of energy that may be interpreted as a jet of particles originating from a single hard gluon is still fairly common.

Naïve power counting, with additional gluon radiation suppressed by a factor of  $\alpha_S(M_V) \approx 0.1$ , is borne out by more detailed calculations, as illustrated in Figure 1. The cross section for producing a  $W$  boson together with jets

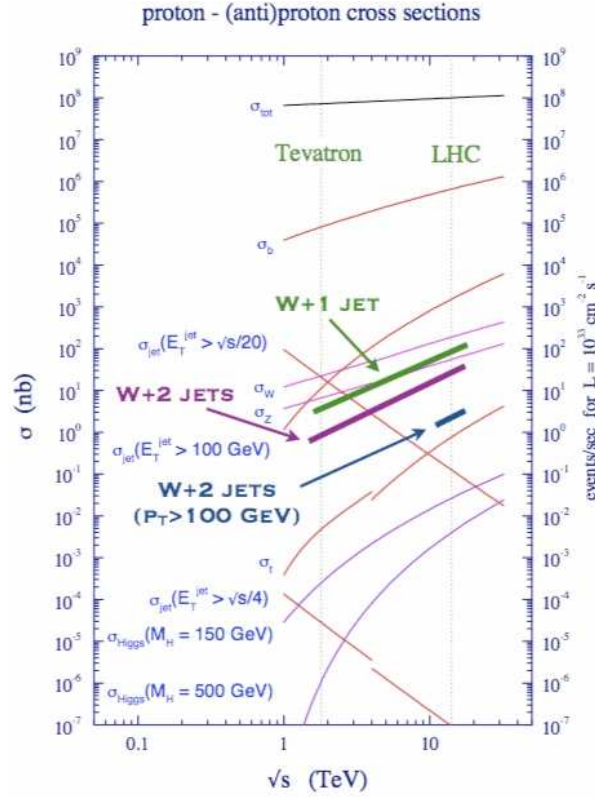


Figure 1: Cross sections for a range of basic processes as a function of the center of mass energy,  $\sqrt{s}$  (adapted from Ref. [1]). The lines indicated for  $W + 1,2$  jets correspond to jets with  $p_T > 20$  GeV and  $|\eta| < 2.5$ .

of  $p_T > 20$  GeV drops by an order of magnitude as each additional jet is considered. The event rate for producing a  $W$  and two such jets is still three orders of magnitude larger than the expected total cross section for production of a Higgs boson of mass 150 GeV. At the LHC, events with two very hard jets, of transverse momenta larger than 100 GeV, will still be relatively commonplace. With a cross section of a few nanobarns, they will be produced as frequently as the much softer  $W + 2$  jet events currently are at the Tevatron.

When folding in the decays of the  $W$  and  $Z$  bosons, one sees that these types of events can lead to substantial backgrounds to many searches at the Tevatron and the LHC. The leptonic decays of the  $W$  and  $Z$  lead to signatures consisting of missing energy, lepton(s) and jets. The presence of a lepton is especially important since it can be used to tag the event and thus it is in such decay modes that these channels are most important. The recent evidence for single top production at the Tevatron [2, 3] has relied crucially on a good understanding of the  $W + 2$  jets background when one or both of the jets may contain bottom quarks. Furthermore, supersymmetry and other models for new physics provide a plethora of signals identified via missing energy and high transverse momentum jets. To pick just one example, a relatively light Higgs boson produced in the vector boson fusion (VBF) channel may be identified via its decay to tau leptons. The observed final state,  $\tau^+\tau^- + \text{jets}$ , is easily mimicked by  $Z$  boson production [4].

In order to have good control of these backgrounds across a wide range of detector acceptances, it is crucial that they are both well understood and accurately simulated theoretically. This goal can only be achieved by systematic advances on two fronts: improvement of the theoretical calculations and validation of calculational tools against existing data. As well as the immediate rewards that may be reaped from such a programme, in the longer term these studies can serve as benchmarks for similar future analyses. In particular, plentiful production of top quark pairs at the LHC will likely necessitate systematic studies of  $t\bar{t} + \text{jet}$  events along similar lines.

## 2. W/Z + JETS

Until recently, the theoretical tools available for comparison with hadron collider data have fallen into two camps: fixed order parton-level calculations and parton shower approaches based on the factorization properties of QCD amplitudes.

Working in fixed order in perturbation theory is by now straightforward at leading order (LO), but already limited at next-to-leading order (NLO) [5]. At next-to-next-to-leading order (NNLO) very few calculations have been performed, with existing ones limited to final states containing only one particle. In addition, the capability of such calculations to model the final states found in the experiments is very limited. At LO each parton in the calculation corresponds to a single jet, with one additional parton per jet allowed for each successive order. Moreover, the number of final state particles that can currently be included in a full NLO calculation is three. Despite these flaws, the great advantage of these calculations is that they provide a reliable normalization of the cross section (NLO and beyond) whose uncertainties can be sensibly estimated (starting at NNLO).

On the other hand the parton shower approach is ideally suited to simulating final states similar to those found in the data which allows, for example, a full analysis of detector effects. This is possible thanks to the additional radiation that is generated stochastically in the shower, after starting from a given (usually 1 or 2 particle) hard final state. The theory underlying such an approach is based on the factorization of the perturbative matrix elements in the limits of soft and collinear partons. Although the effects of such soft physics are taken into account and well-modelled, hard or large angle radiation is in general poorly described. In addition, this approach is generally implemented only using LO matrix elements, limiting the trustworthiness of the predictions accordingly.

Recently there has been progress on both fronts. At fixed order there have been many developments aimed at automating NLO calculations so that they may be applied to higher multiplicity final states [5, 6]. Regarding parton showers, there have been two major developments in recent years. The first, generically known as *matching*, supplements the traditional parton shower by including hard matrix elements with more partons. In this way, the poor approximations usually employed by a parton shower in hard configurations are eliminated by using the exact matrix elements. The original implementations of such schemes [7, 8] have recently spawned further versions [9, 10, 11], although the basic principle remains the same. The arena of  $W/Z + \text{jet}$  simulation has been an ideal testing ground

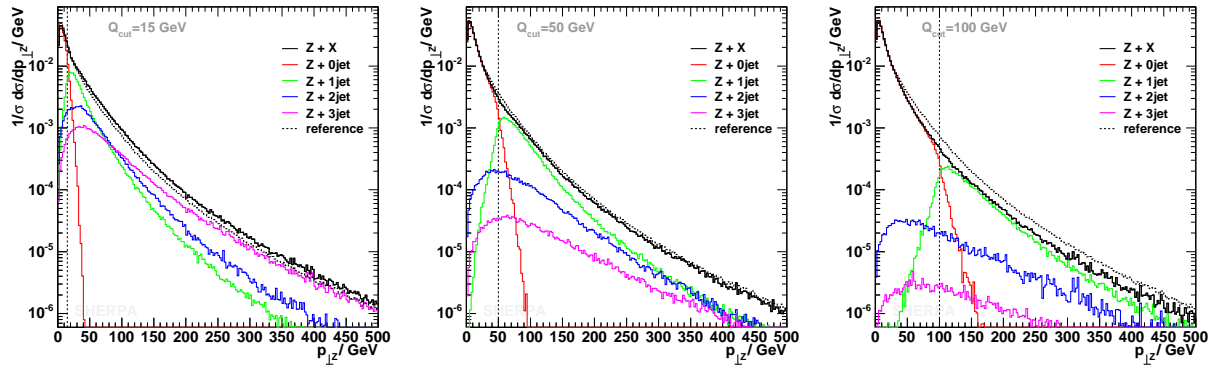


Figure 2: The  $Z$  transverse momentum distribution, taken from [14].

for the idea of matching in parton showers, as we shall discuss shortly. Finally, a major advance in this area has been the inclusion of NLO matrix elements in a parton shower [12]. At present this implementation does not address the issue of  $W/Z$ +jet production, although it is not infeasible in the near future.

Whichever matching procedure is used, an artificial parameter must be introduced that serves to discriminate between the regions of hard matrix elements and of collinear approximations. Although predictions should be largely independent of this parameter, if it is not chosen appropriately then the hard matrix elements may not be used when necessary and the inadequacy of the original (unmatched) parton shower returns. As an example of the matching procedure applied to  $Z$ +jet production, we refer to the CKKW scheme as implemented in the SHERPA [13] event generator. In this case the parameter is the jet resolution cutoff,  $Q_{\text{cut}}$ , and a study of its impact upon the inclusive  $Z$   $p_T$  spectrum is shown in Figure 2. The contributions from each set of hard matrix elements are shown separately, with the upper curve obtained from their sum. This curve is not only smooth, but relatively stable with respect to the change from  $Q_{\text{cut}} = 15$  GeV to  $Q_{\text{cut}} = 50$  GeV. However, for  $Q_{\text{cut}} = 100$  GeV the distribution begins to fall off more rapidly at large  $p_T$ , indicating that the hard matrix elements are not being effectively utilized.

As well as such studies of individual generators, recently much work has gone into comparisons of the showering algorithms and matching procedures that are in general use. The results show that all generators are broadly consistent, with differences that can mostly be accounted for by the usual variation of scales [15]. Most importantly, the input parameters can be tuned with Tevatron data and then the results extrapolated for use at the LHC.

On the side of fixed-order calculations, the pace of developments that *directly* confront experimental data has been somewhat slower. The NNLO corrections to inclusive rates and distributions for  $W$  and  $Z$  production have been known already for five years [16]. This accuracy, leaving residual uncertainties of a few percent, is highly desirable – for example, in order to pin down PDFs or possibly even to serve as a luminosity monitor [17, 18, 19]. The NLO calculation for a final state involving one additional jet was performed a long time ago [20], with the corrections to  $W/Z + 2$  jet observables known at the same order more recently [21]. The prospect of a NNLO calculation of  $W/Z + 1$  jet production has become more imminent in the last year, thanks to a pioneering effort to complete the NNLO corrections to the 3-jet rate in  $e^+e^-$  collisions [22]. The two processes are related by crossing and, although it is non-trivial to address the difficult issue of doubly-infrared singularities in the hadron collider environment, a clear pathway between the two exists. For now an extension of this approach to tackle the  $W/Z+2$  jet calculations at NNLO appears very unlikely and further calculational advances are required. Beyond two jets, fixed order calculations are limited to leading order at present, although this may soon be addressed as one of the plethora of approaches aimed at performing arbitrary multi-leg processes at 1-loop order matures<sup>1</sup>. However, for now one must choose between either higher orders in QCD or a parton shower.

<sup>1</sup> Indeed, there has been progress towards a calculation of  $W/Z + 3$  jets at NLO since this symposium [23]

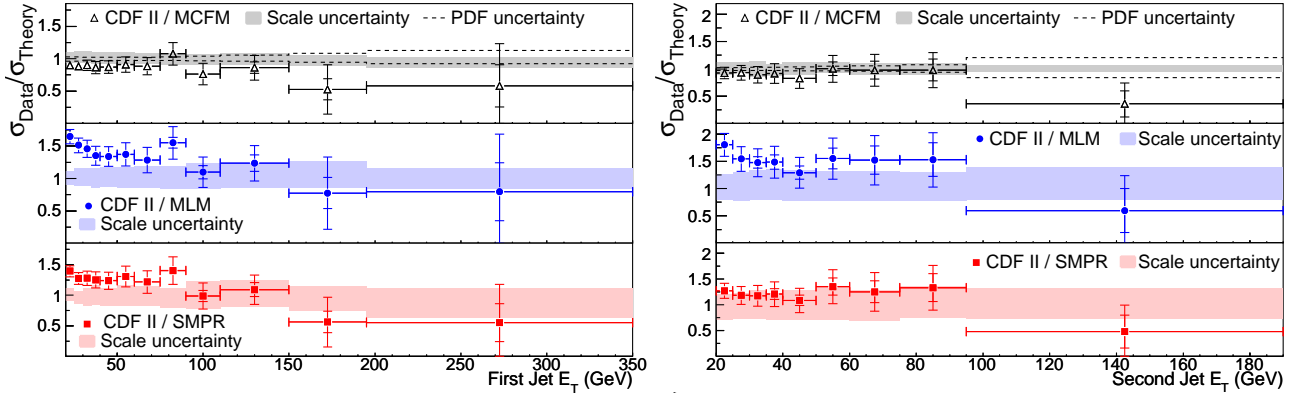


Figure 3: The  $E_T$  of the first (left) and second (right) jets in  $W$ +jet events, adapted from Ref. [24]. The CDF data are normalized to NLO predictions (top), ALPGEN+HERWIG with the MLM matching prescription (middle) and MADGRAPH+PYTHIA using the CKKW procedure (bottom).

A recent comparison of CDF  $W$ +jet data with a variety of these approaches is shown in Figure 3 [24]. It appears that, at least for the first two jets, the parton shower approaches do not reproduce the shape of the  $E_T$  distributions very well. On the other hand, the NLO calculation tracks the data reasonably successfully, with deviations largely captured by the combination of scale and PDF uncertainties. Of note is the omission of comparisons of the third and higher jets; as already noted, for these no NLO prediction is currently available and we must be content with qualitative agreement with the parton shower approaches.

In light of this comparison, several open questions remain. The NLO description appears to excel, but the agreement could be considered too good. At the lowest values of  $E_T$  considered one might expect a fixed order calculation to be less accurate, but in fact this is precisely where the NLO calculation agrees best and the parton showers fare worst. At this point it is instructive to recall that the NLO calculation cannot account for the effects of hadronization or the underlying event. The lack of such information means that the quality of the comparison may be somewhat misleading. This question cannot be satisfactorily answered until a parton shower is developed to NLO accuracy for this final state. We can hope that, by the time that LHC analyses have reached a similar stage, such a tool is available. A more immediate diagnostic of the fixed order and parton shower approaches could be provided by a detailed comparison of predictions for observables other than the jet transverse momenta. These may indicate areas where the theoretical predictions perform particularly badly and could be used to assess the degree to which the NLO agreement might be accidental.

At the LHC similar studies will no doubt be performed. It is interesting to note that related processes, such as  $WW$  production, despite being suppressed by a factor of 1,000 compared to the single  $W$  process, still have cross sections in the region of 100 pb. The rates for emitting additional radiation will again be suppressed by factors of order 0.1 for each additional jet. When combined with the large luminosities expected at the LHC, this means that plenty of  $WW$ +jet events will be produced. Such events form an irreducible background to various searches for the Higgs boson when its mass is large enough for the decay  $H \rightarrow WW^*$  to be significant, notably in the gluon fusion and vector boson fusion channels [25]. In the former a jet veto is often applied to reject other backgrounds, thus focussing on the  $WW + 0$  jets final state. In the latter one expects the Higgs boson to be produced centrally, accompanied by two very forward jets. Either both or just one of these jets may be tagged (to increase the expected signal cross section), thus requiring a good knowledge of  $WW + 1, 2$  jet observables. One could therefore imagine that the breadth of experience accumulated in  $W$ +jet studies could profitably be applied to provide a systematic analysis of  $WW$ +jet backgrounds. On the theoretical side, an extra vector boson means that parton shower (plus matching) approaches will require more computational effort, but no new developments. On the side of fixed order, it is quite a different story. NLO corrections to inclusive  $WW$  production have long been known [26, 27, 28], with corrections to the emission of a single hard jet only very recently computed [6, 29, 30]. Again, the promised advances

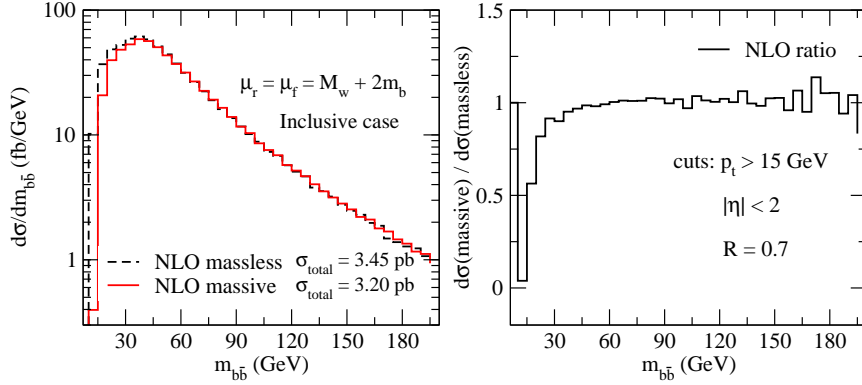


Figure 4: Left: A comparison of two NLO calculations of  $Wb\bar{b}$  production at the Tevatron, including the effects of the  $b$ -quark mass (solid red) and neglecting them (dashed black). The cross section is presented as a function of the  $b\bar{b}$  pair invariant mass,  $m_{b\bar{b}}$ . Right: the ratio of the massive and massless calculations, again as a function of  $m_{b\bar{b}}$ .

in multi-leg NLO computations could have an immediate impact in this arena.

### 3. W/Z + HEAVY FLAVOR

Although the production of heavy flavors (in particular, bottom and charm quarks) and the production of light quark jets share many similarities, at a theoretical level they differ in a crucial way. For massless quarks, the matrix elements diverge in the soft and collinear limits. This means that predictions diverge, for example, in the limit that  $p_T(q) \rightarrow 0$  so that a minimum jet  $p_T$  and angular separation must be imposed in order to compare with similarly-restricted experimental data. For a massive quark, its mass  $M$  serves to regulate the divergence, which instead behaves as  $\log(M^2)$ . Therefore no cuts are necessary in order to properly define the cross section theoretically, so that it can then be calculated inclusively. Unfortunately, this appealing feature comes at a price: including the mass renders the fixed-order calculations more difficult, which is particularly troublesome when already at the limit of current sophistication – namely, a vector boson plus two partons.

Of course, some analyses do not demand close attention to the low transverse momentum behavior of these cross sections. In that case it can be sufficient to work in the approximation that the heavy quark is massless. The applicability of such an approximation is illustrated well by the comparison between two calculations of  $Wb\bar{b}$  production, as shown in Figure 4 [31]. When applying typical jet cuts, the most recent calculation that includes the mass of the  $b$ -quark differs from an earlier one for  $m_b = 0$  by less than 10% in the total cross section (i.e. a difference of order  $m_b^2/p_T^{\min}(b)^2$ ). As seen in the right panel, this deviation is due entirely to the behavior in the threshold region that cannot possibly be captured correctly in the massless calculation. Away from threshold the two agree.

In some instances one cannot simply restrict the phase space in this fashion. In particular, it is often advantageous to tag only one heavy quark in order to improve experimental efficiency (see for example Ref. [32]). In such a scenario the partner anti-quark must be integrated over the whole of the phase space region so that the straightforward massless approach cannot be used. However another technique is suitable in exactly this limit, one that is based on the use of heavy quark parton distribution functions.

At very high center-of-mass energies we are used to the idea that we may become sensitive to heavy quark content of the (anti-)proton sea. In fact, we are well used to this approach in the context of  $W$  production, where the contribution shown in the left diagram of Figure 5, dependent on the charm quark distribution, is usually included when calculating the inclusive  $W$  cross section (and is responsible for about 5% of the total at the LHC). The bottom

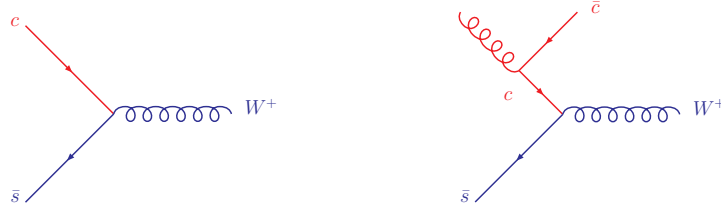


Figure 5: Left: Production of a  $W$  boson via the charm quark distribution function, with an implicit anti-charm quark integrated out. Such a diagram is included in the “variable flavor scheme” (VFS) when the anti-charm quark is not relevant to the analysis. Right: Production of a  $W$  boson and an anti-charm quark via an initial state gluon splitting,  $g \rightarrow c\bar{c}$ . If the anti-charm quark is subsequently integrated out, this corresponds to the “fixed flavor scheme” (FFS).

quark distribution is a little less familiar, with its use at the Tevatron mainly limited to studies of  $t$ -channel single top production. However it will be more important at the LHC due to the increased role of the proton sea. The heavy quark distribution function can be thought of as folding back a gluon splitting into heavy quark pairs, from the matrix elements and into the PDF. This splitting is shown explicitly in the right-hand diagram of Figure 5. The calculation of these two processes, in the so-called variable flavor scheme (VFS, left) and fixed flavor scheme (FFS, right), should therefore agree at some level when information about the additional massive anti-charm quark in the FFS is integrated out.

The two approaches are of course exactly equivalent in the full theory. At a given order of perturbation theory this is not the case and, in general, they do not agree particularly well at leading order. Each approach has its own advantages and weaknesses, which are summarized in Table I. In that light it is important to understand what

Table I: A comparison of the features of calculations performed in the variable and fixed flavor schemes, adapted from [33].

Feature	Variable (heavy quark PDF)	Fixed (massive ME)
Recoiling heavy quark	no	yes
Exact kinematics	no	yes
Higher order logs	yes	no
Easier NLO	yes	no

differences exist in practice and if (or when) one approach is superior. In particular, parton showers typically make use of the fixed flavor scheme (for which it is also easier to perform matching) so that the choice between the two schemes is usually quite stark.

### 3.1. $W$ + charm

This is the simplest possible process for which the two schemes can be compared [34]. On the experimental side, the rate of events is large and the final state is relatively easy to identify. On the theoretical side, as previously noted, the VFS simply corresponds to the  $2 \rightarrow 1$  Drell Yan process (Figure 5, left) that is known to NNLO whilst the FFS calculation is a  $2 \rightarrow 2$  scattering (Figure 5, right) that is known at NLO [35].

Comparing the two calculations at LO, one finds that choosing the “natural” scale of the process, the  $W$  mass (for both renormalization and factorization scales), the cross section in the VFS is twice as large as in the FFS [36]. However both calculations exhibit strong scale dependence and, if one were to choose a small scale in the region  $(0.15 - 0.3)m_W$ , the two would agree at the 20% level or better. Such a scale choice is well-motivated theoretically, particularly for the VFS. One argument is that a more appropriate scale for the gluon splitting that produces the charm quark is the charm quark mass, so that a better single scale choice is the geometric mean,  $\sqrt{m_c m_W}$ . Alternatively, one can argue that the collinear approximation used in deriving the heavy quark PDFs is only valid up to much smaller scales [37, 38]. In any case, the VFS result is largely independent of the choice of scale at NLO.

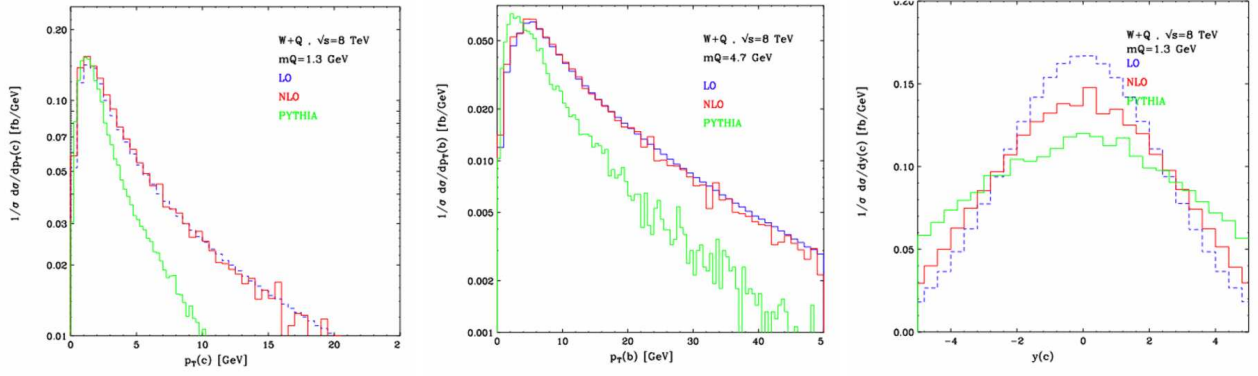


Figure 6: Left: The  $p_T$  of the charm quark predicted by the FFS (upper two curves, overlapping) and in Pythia (lower curve). Middle: The same quantity, but with the charm mass inflated to 4.7 GeV in order to probe the kinematics involved in bottom quark production. Right: The rapidity of the charm quark (FFS LO, upper; NLO, middle; Pythia, lower). All curves are normalized to the corresponding predicted total cross section, so represent shapes only. Taken from [36].

As previously noted, since the VFS calculation yields approximately 5% of the inclusive  $W$  cross section at the LHC, differences of this size between the two calculations bring into question claims of a few percent accuracy of the theoretical prediction. Even more important is the issue of kinematical distributions, which are usually simulated using a parton shower. A comparison between the predictions of the FFS (at LO and NLO) with PYTHIA is shown in Figure 6, for the transverse momentum and rapidity distributions of the charm quark. Firstly, it is clear from this figure that the prediction for the shape of the transverse momentum distribution of the charm quark in the FFS does not change from LO to NLO. The lack of substantial corrections to this distribution at large  $p_T(c)$  seems to indicate that there are no large logarithmic corrections of the sort that purport to be resummed in the VFS. The PYTHIA prediction for the same quantity of course lacks proper inclusion of hard radiation, leading to a large underestimate at high  $p_T(c)$ . It is interesting to note that the region of small transverse momentum appears to agree with the FFS calculation (the left pane). However, when the charm quark mass is increased in order to probe the kinematics involved in bottom quark production (middle pane), the position of the peak at low  $p_T$  does not agree with the FFS curves. The underlying cause of this disagreement is still under investigation, but the conclusions could have significant implications for analyses at the LHC. Finally it is heartening to note that, at least for the charm quark, the PYTHIA prediction seems to reproduce one of the features of the NLO calculation, namely a broadening in the rapidity distribution of the charm quark (right pane). There is clearly much room for improvement, particularly as regards to parton shower/matrix element merging, which should shed further light on these comparisons.

Of course, the ultimate arbiter in such disputes is the data itself. A recent result from CDF, with the very basic selection cuts  $p_T(c) > 20$  GeV,  $|\eta(c)| < 1.5$ , yields the result,  $\sigma \times \text{BR}(W \rightarrow e\nu) = 9.8 \pm 3.2$  pb [39]. This is to be compared with the NLO prediction in the FFS of  $\sigma(\text{NLO}) = 11.0(+1.4-3.0)$  pb, where the central result corresponds to a scale of  $Q = 40$  GeV and the theoretical error accounts for variation both over a wide range of scales and over a selection of PDF sets. Thus there is good agreement between experiment and theory, albeit with large residual uncertainty on both sides.

### 3.2. W + bottom

This channel is not a direct analogue of  $W + c$  production, due to CKM suppression. Instead, a hard bottom quark must be produced together with other (undetected) particles. The two leading mechanisms for producing such final states are shown in Figure 7. The left pane shows usual  $Wb\bar{b}$  production (in the FFS), where we assume that only one of the quarks is tagged. As discussed above, this requires a calculation including the bottom quark mass, which is known at NLO [31]. The right pane shows production of the state  $Wbq$  (with  $q$  a light quark) in the VFS, which is also known at NLO [40]. By demanding that the  $b$  quark is observed, no singularities are encountered when



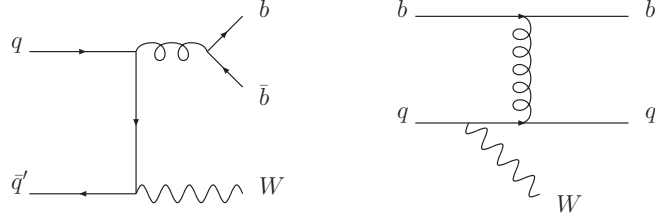


Figure 7: Left: LO diagram for production of a  $W$  and two  $b$ -quarks in the FFS. Right: LO diagram for production of a  $W$ , a  $b$  quark and a light quark (VFS).

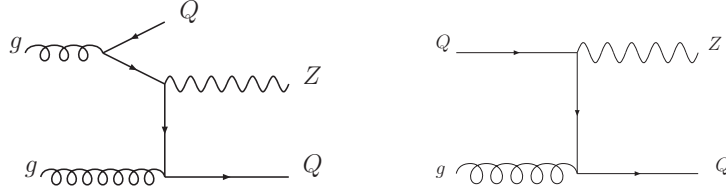


Figure 8: Left: LO diagram for the process  $gg \rightarrow ZQ\bar{Q}$  (FFS). Right: The equivalent process using a heavy quark PDF (VFS).

evaluating this contribution. Note that the NLO corrections to the corresponding FFS process, a  $Wb\bar{b}q$  final state, are not known.

To obtain the best possible prediction for the rate of  $Wb$  production, both of the above contributions can be combined. Care must be taken to ensure that no overlap between the calculations is included, but some preliminary results are already available [41]. The relative importance of the two contributions varies with the collider: at the Tevatron the valence quarks ensure the  $Wb\bar{b}$  process dominates, whereas at the LHC the gluon (and hence  $b$ -quark) flux means the  $Wbq$  process is largest. Early results indicate that, for typical cuts, the total cross section is enhanced by a factor of about 1.5 at both machines. Although this is a substantial increase it is not enough to explain preliminary results from CDF [42] that indicate a shortfall of about a factor of three or four when comparing LO theory to data. This is especially intriguing since the shapes of distributions are reasonably well described. At this stage it seems that the NLO corrections may be part of the solution, but are unlikely to solve the puzzle completely.

### 3.3. $Z$ + heavy quark

The production of a  $Z$  boson in association with either a bottom or charm quark is somewhat analogous to  $Wb$  production. The additional complication is that the FFS now also receives a contribution from  $gg$  initial states at LO, which can alternatively be described by a simpler VFS calculation (see Fig. 8). A comparison of the two approaches (at LO) is shown in Fig. 9, using the cuts  $p_T(b) > 20$  GeV,  $|\eta(b)| < 2.5$  at the Tevatron. There are substantial differences between the two calculations, which are somewhat reduced for a smaller scale choice. Now that the NLO corrections to both processes are known [43, 44], real understanding may follow from a rigorous comparison of the two. The observed discrepancy is in marked contrast to the (similar) process in which a photon and bottom quark are observed in the final state. In that case the scale of the problem is simply set by the photon  $p_T$  and a comparison of the two schemes at LO already shows good agreement [33].

Once again, experimental data has the chance to discriminate between the two calculational schemes. However, information on the total rate obtained by the D0 experiment [45] does not immediately yield the desired information. The ratio  $\mathcal{R} = \sigma(Z + b\text{-jet})/\sigma(Z + \text{jet})$  is measured to be  $\mathcal{R}_{\text{exp}} = 0.021 \pm 0.004$ , to be compared with the predictions,  $\mathcal{R}_{\text{VFS,NLO}} = 0.018 \pm 0.004$  and  $\mathcal{R}_{\text{FFS,LO}} = 0.015\text{--}0.023$ . The reason for the good agreement between the two calculations can be found in Fig. 9. It is clear that the  $q\bar{q}$  contribution, present in both schemes, dominates the total cross section. It is only by comparing the  $p_T$  distribution of the  $Z$  below about 40 GeV that the two schemes could be probed. It is essential that such analyses are performed in order to better understand the theoretical predictions.



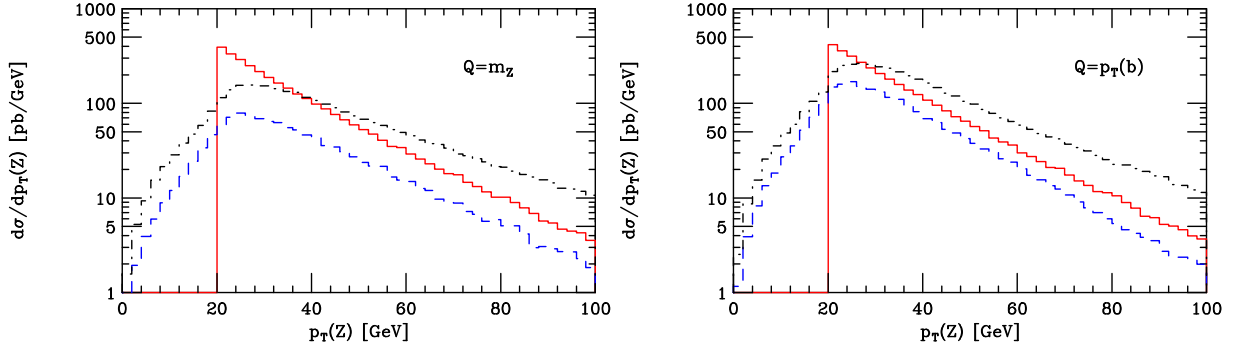


Figure 9: The transverse momentum distribution of the  $Z$  boson produced in  $Z + b$  final states as calculated in the FFS and VFS using a high scale  $Q = m_Z$  (left) and a much smaller one,  $Q = p_T(b)$  (right). The VFS (red, solid) curve is zero in the region below the kinematic limit,  $p_T(Z) < 20$  GeV. The lower (blue, dashed) curve is the FFS result. The upper (black, dot-dashed) curve shows the contribution from the other  $q\bar{q} \rightarrow Z b\bar{b}$  process that is common to both schemes.

Table II: Theoretical activity in  $W/Z +$  heavy flavor in recent years.

	1 $c$ -tag	1 $b$ -tag	2 $c$ -tags	2 $b$ -tags
$W + 1$ jet	FFS NLO [35, 46]	FFS+HVQ NLO [41]	n/a	n/a
$W + 2$ jets	LO only	HVQ NLO [40]	FFS NLO [31]	
$Z + 1$ jet	FFS NLO [44], HVQ NLO [43]		n/a	n/a
$Z + 2$ jets	HVQ NLO [47]		FFS NLO [44]	

## 4. OUTLOOK

In the absence of heavy quarks, the situation is quite encouraging. There is good agreement between the latest experimental results and theoretical predictions. The technique of matrix element matching in a parton shower is by now a mature field and, if anything, could benefit from more experimental measurements with which to compare and tune. With regards to NLO, results for  $W$  and  $Z$  production with up to two hard jets agree well with data, but cannot address higher-multiplicity final states. However a host of new automated multi-leg approaches should fill this need in the near future.

When considering heavy quark production, the picture is not so clear. In some cases (notably  $Wb$  production) theoretical predictions fail to reproduce the rates observed experimentally; elsewhere agreement is patchy at best. However it is heartening to see that data from the Tevatron is now seriously confronting the two theoretical approaches commonly used in this arena, the FFS and VFS. On the theoretical side there has been a wealth of activity in recent years, summarized in Table II. This opens up a real chance to make a systematic evaluation of these calculations, both with each other and against data, in order to properly interpret the theoretical tools.

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